Continuous Random Variables

Lecture 22
Section 7.5.4

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Outline

1. Homework Review
2. Random Variables
   - The Uniform Distribution
   - A Non-uniform Distribution
3. Hypothesis Testing
   - Sample Size 1
   - Sample Size 2
4. Assignment

Homework Review

Exercise 6.25, page 380.

- Machine A makes parts whose lengths are approximately normally distributed with a mean of 4.6 mm and a standard deviation of 0.1 mm.
- Machine B makes parts whose lengths are approximately normally distributed with a mean of 4.9 mm and a standard deviation of 0.1 mm.
- Suppose that you have a box of parts which you believe are from Machine A, but you're not sure.
- You decide to test the hypotheses H_0: The parts are from Machine A versus H_1: The parts are from Machine B, by randomly selecting one part from the box and measuring it.

Homework Review

Exercise 6.25, page 380.

(a) Draw the distributions for the lengths of parts under H_0 and under H_1. For both sketches, label the x-axis from 4.2 to 5.2 by 0.1. Be sure to include all important features.

(b) Suppose that you get a length of 4.8 mm.

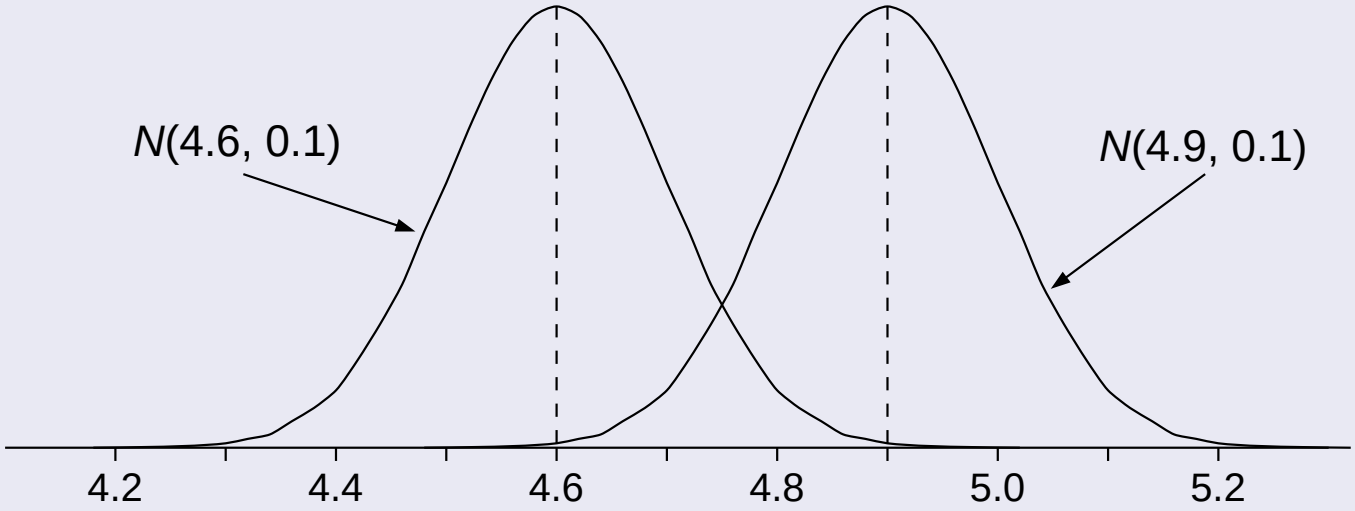
  (i) In your sketch for part (a), shade in the region that corresponds to this p-value and clearly label the region as such.

  (ii) Compute the p-value for your test.

(c) What is your decision at the 0.01 level?

Homework Review

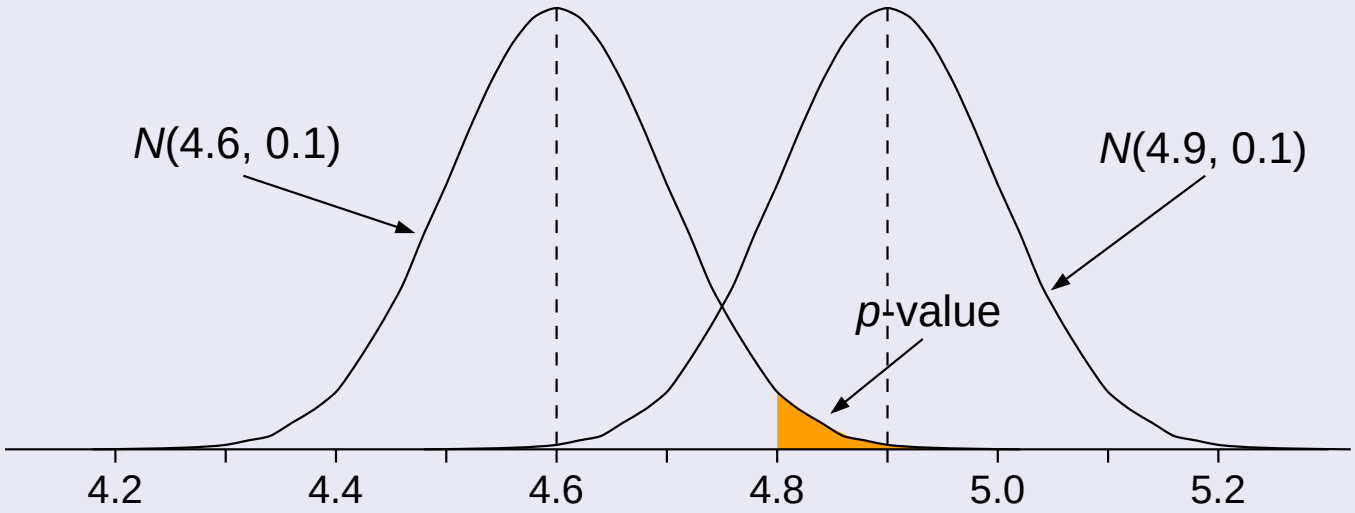
Solution



(a)

Homework Review

Solution



(b) The p-value is 0.0228.

(c) The decision at the 0.01 level is to accept H_0.

Random Variables

Definition (Random variable)

A random variable is a variable whose value is determined by the outcome of a random process.

Definition (Discrete random variable)

A discrete random variable is a random variable whose set of possible values is a discrete set.

Definition (Continuous random variable)

A continuous random variable is a random variable whose set of possible values is a continuous set.

Continuous Probability Distribution Functions

Definition (Continuous Probability Distribution Function)

A continuous probability distribution function, or pdf, for a random variable X is a continuous function with the property that the area below the graph of the function between any two points a and b equals the probability that $a \leq X \leq b$.

- Remember,

$$\text{AREA} = \text{PROPORTION} = \text{PROBABILITY}$$

Example

- The TI-83 will return a random number between 0 and 1 if we enter `rand` and press `ENTER`.
- These numbers have a uniform distribution from 0 to 1.
- Let X be the random number whose value is determined by the `rand` function.

Example

- What is the probability that the random number is at least 0.3?

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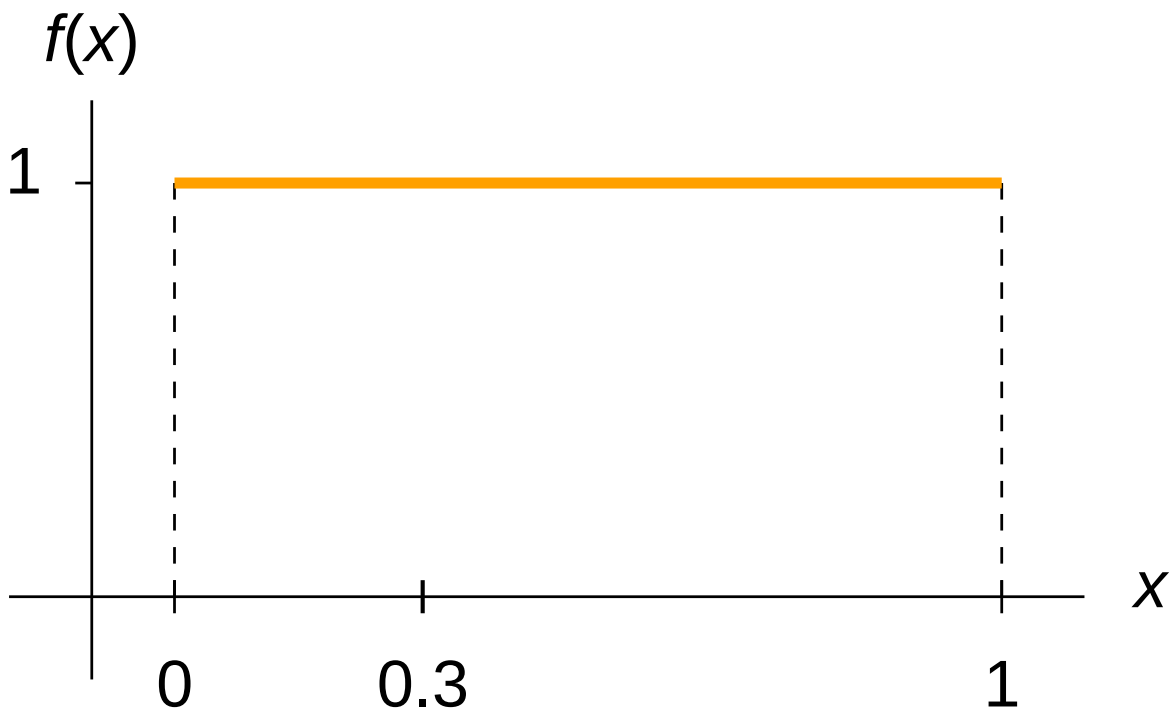

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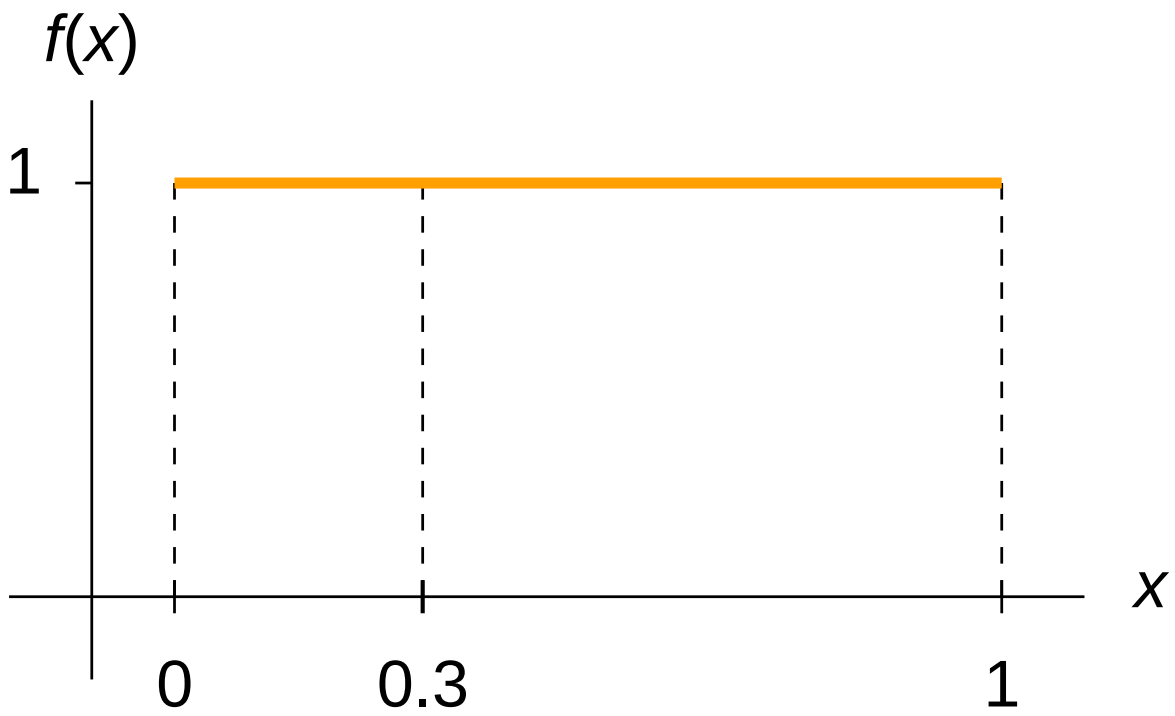

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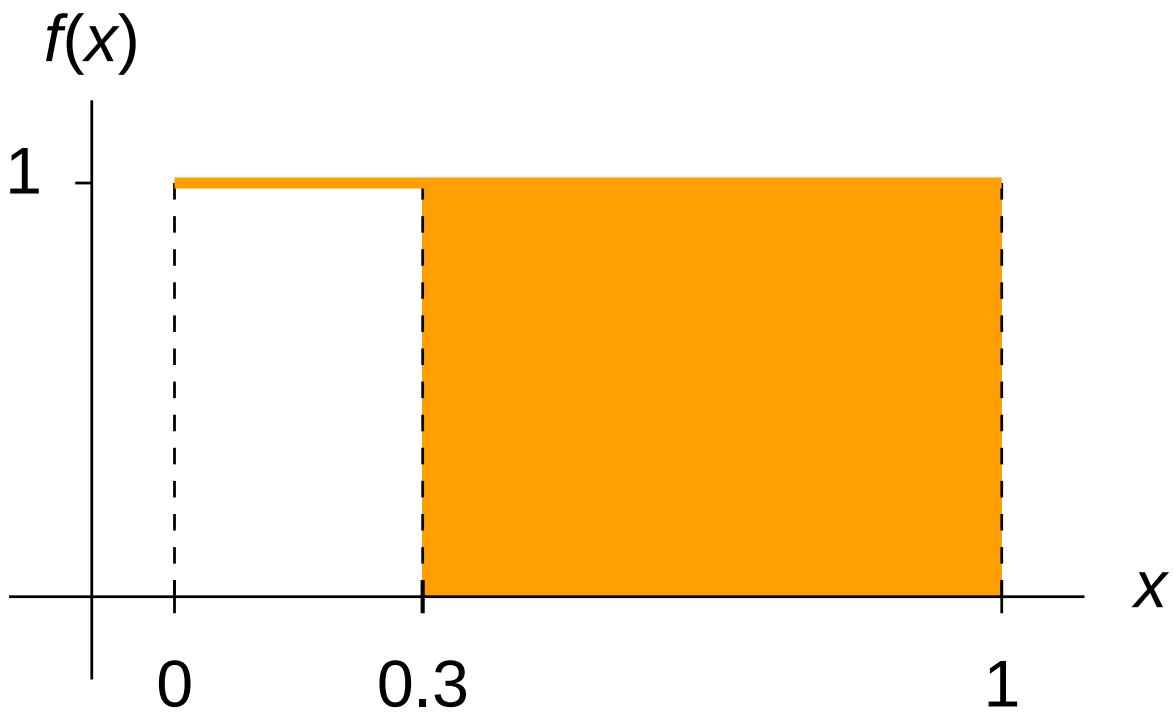

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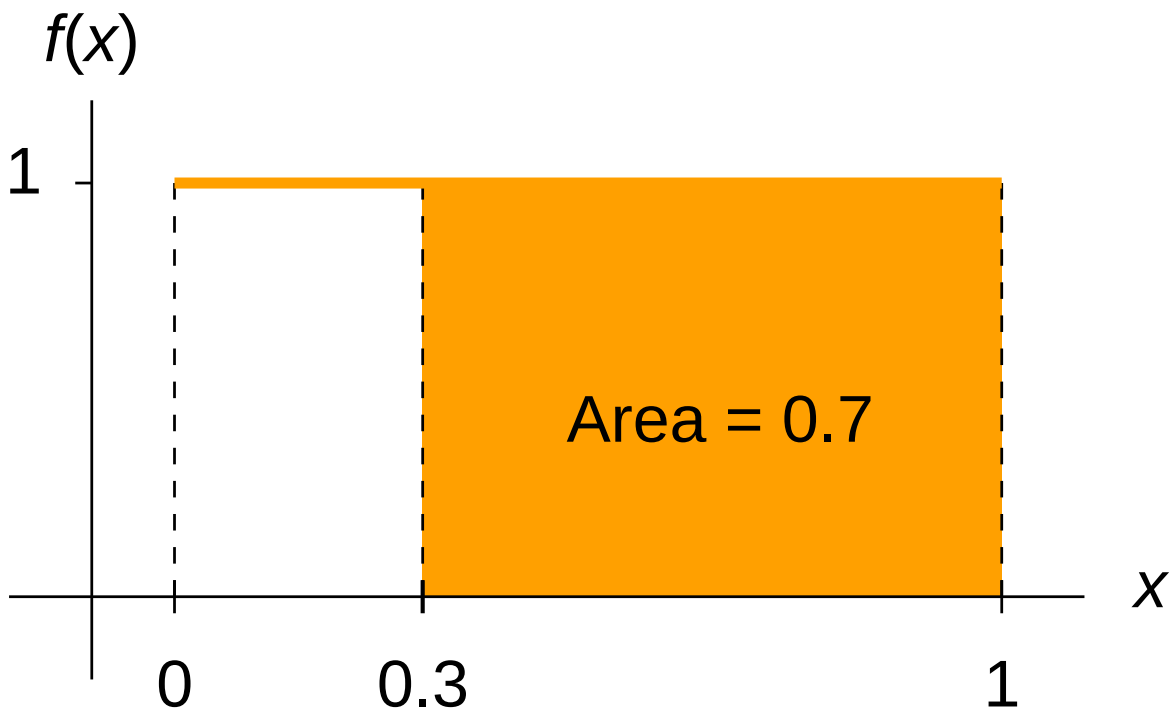

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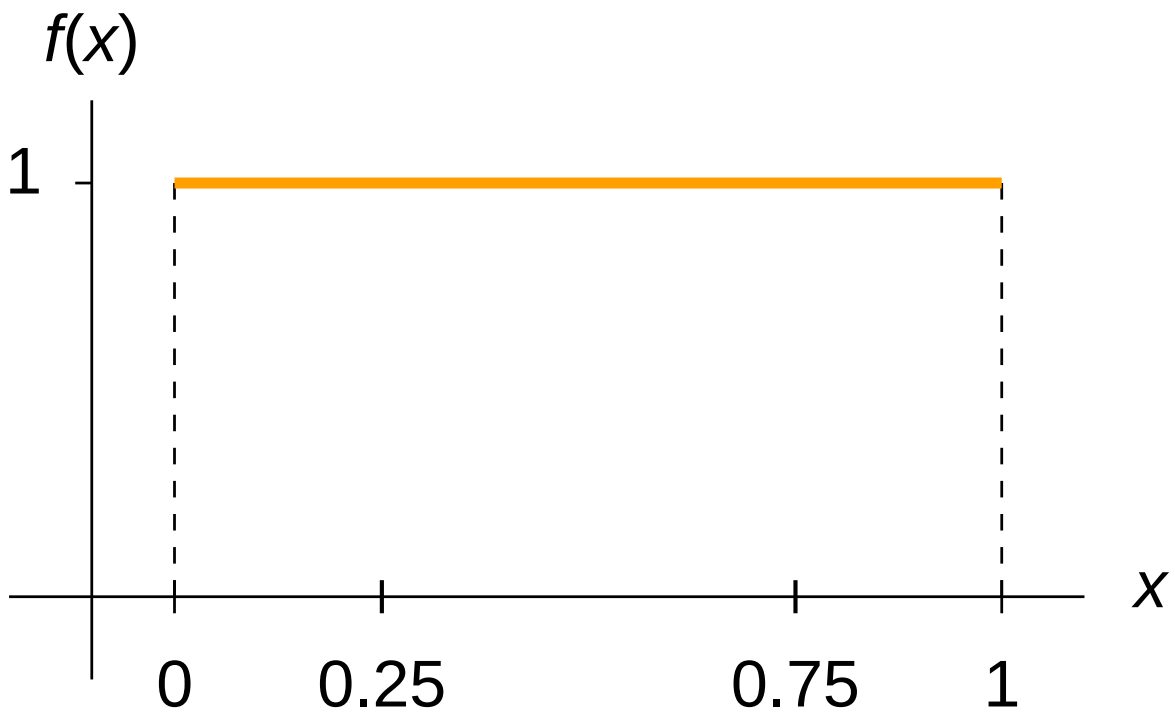

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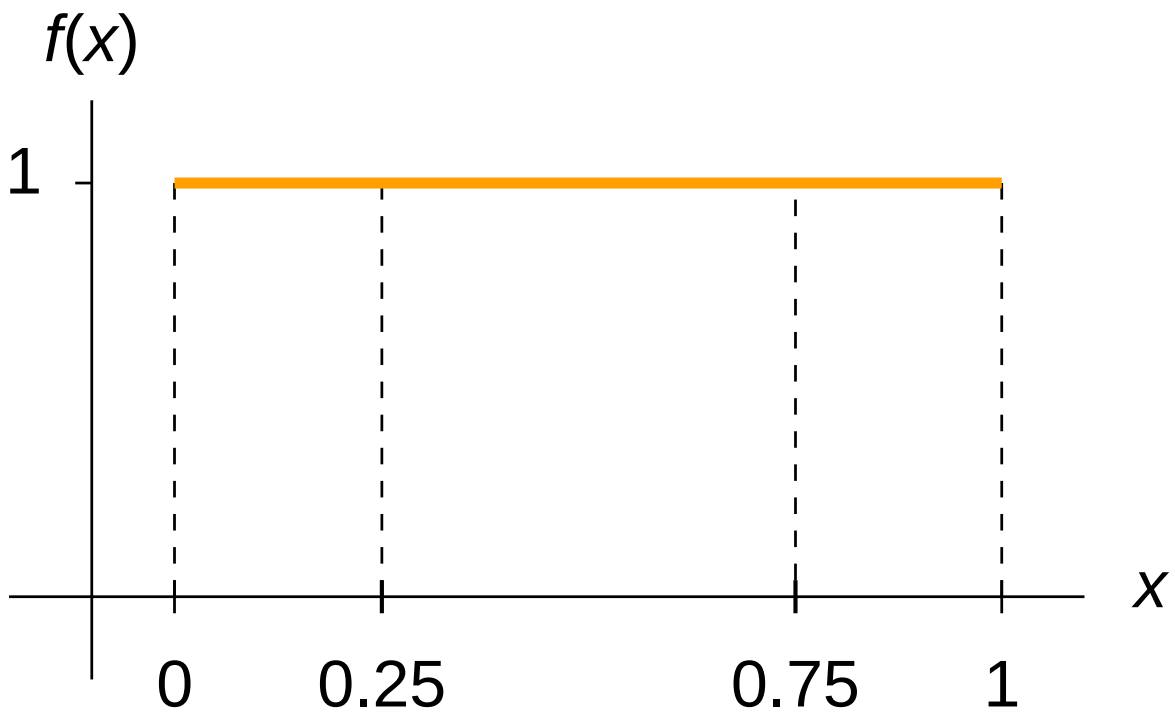

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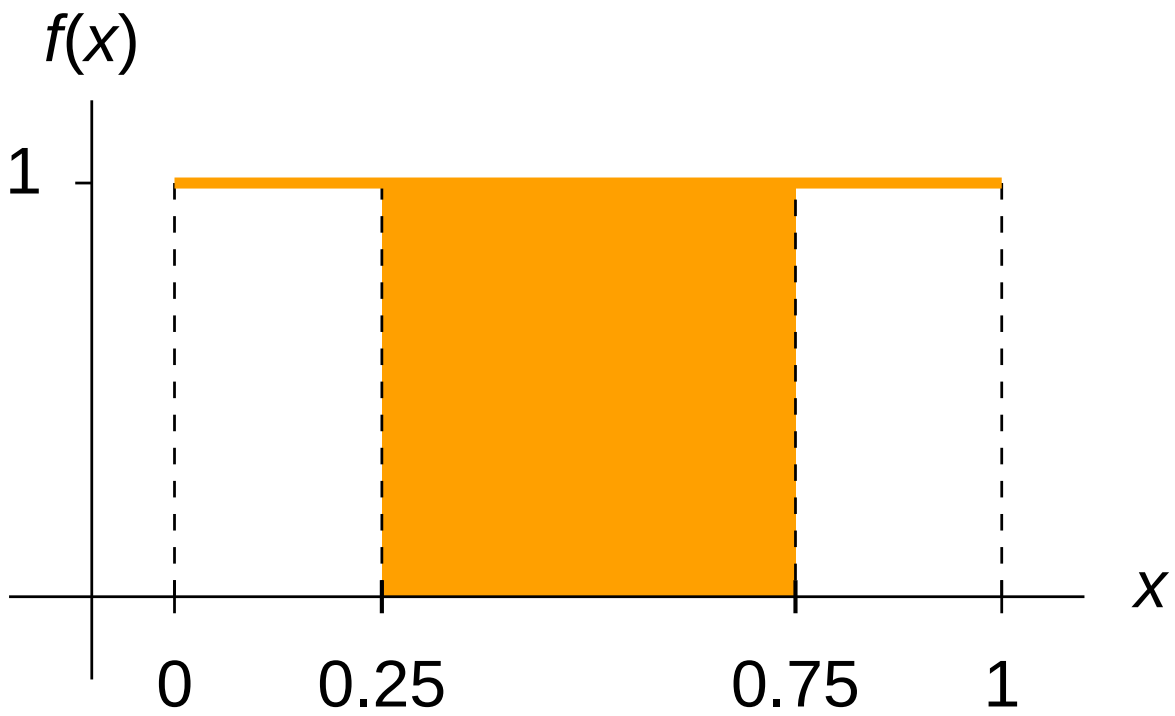

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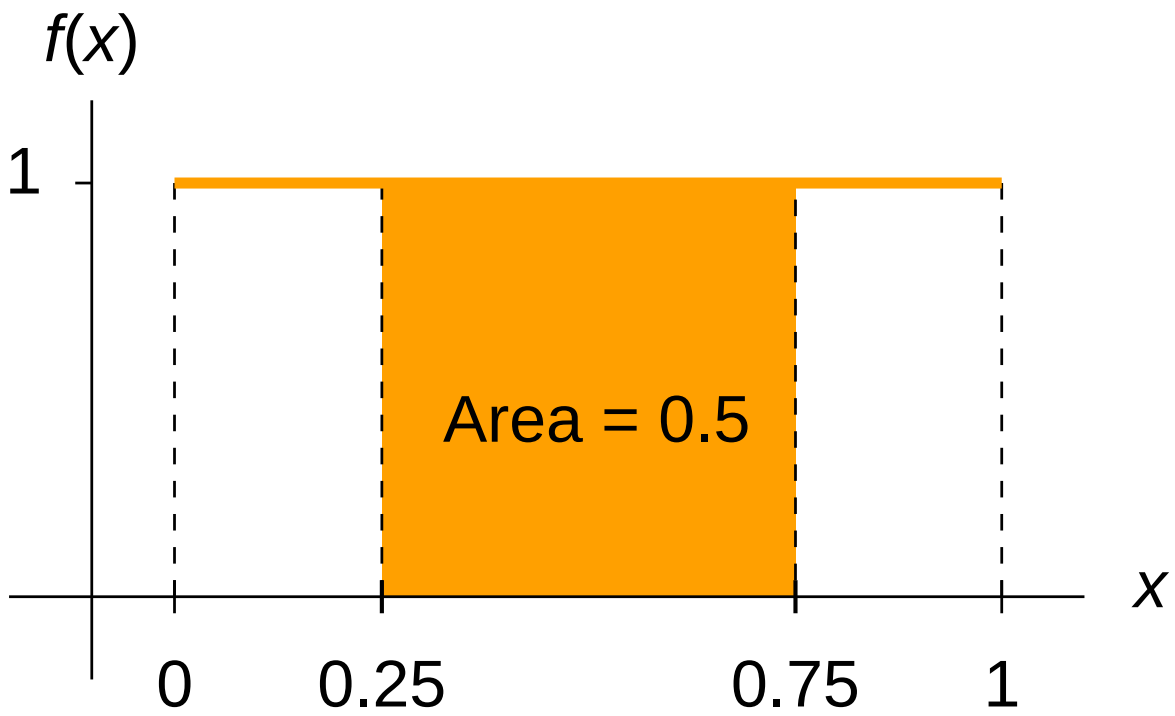
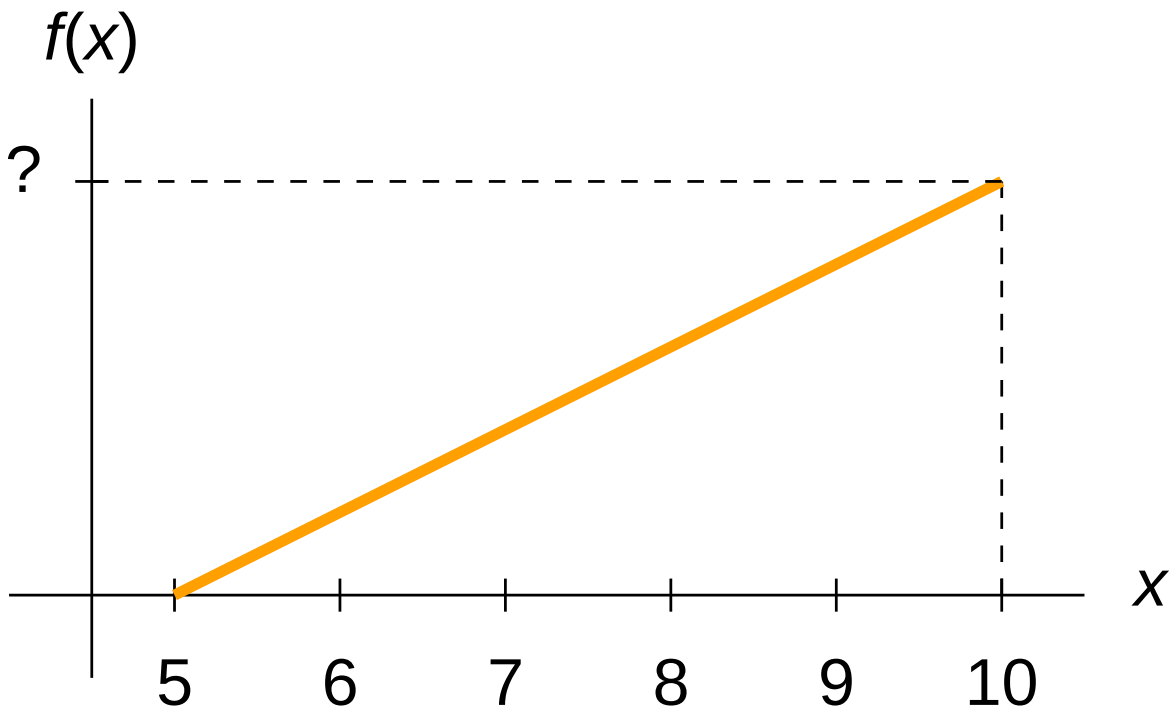


A Non-Uniform Distribution

- What is the height of this distribution?



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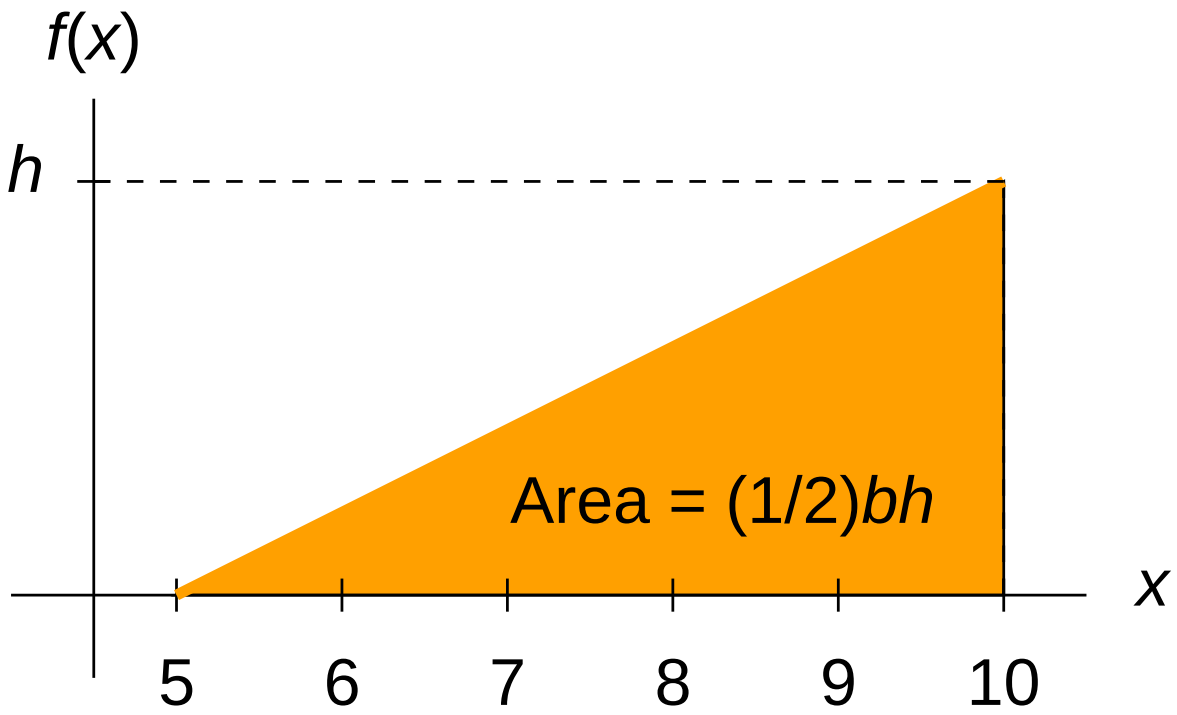

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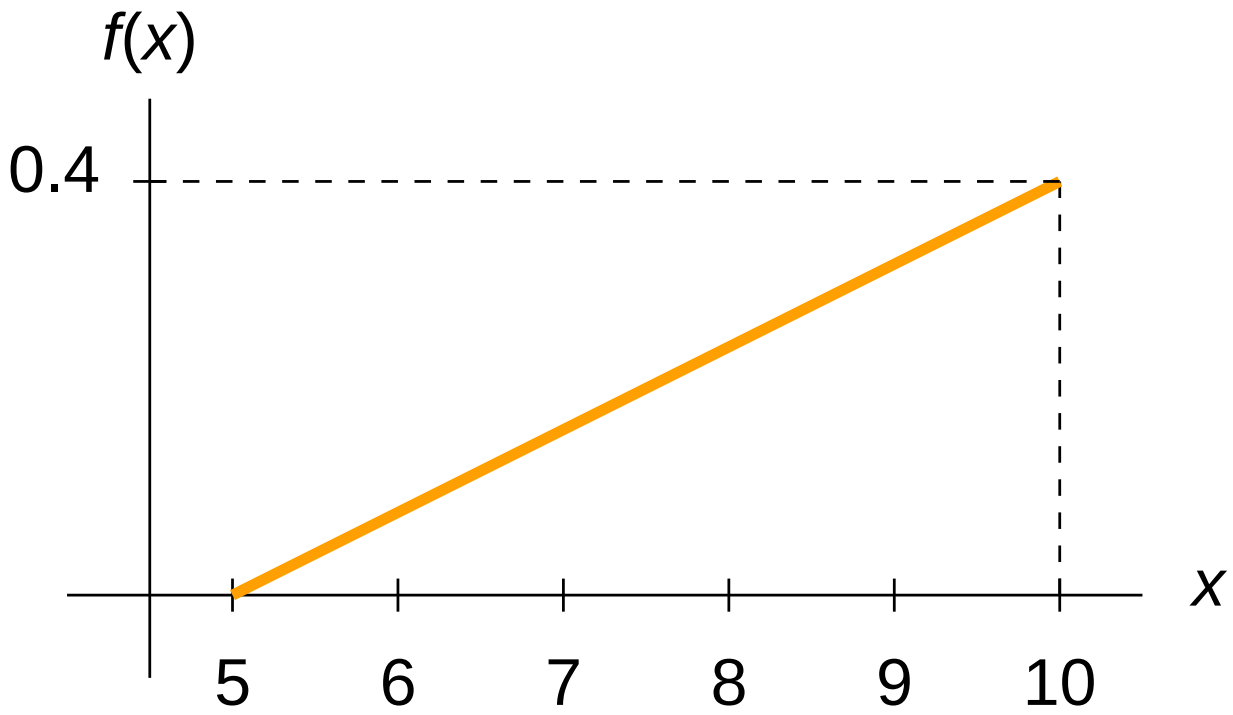
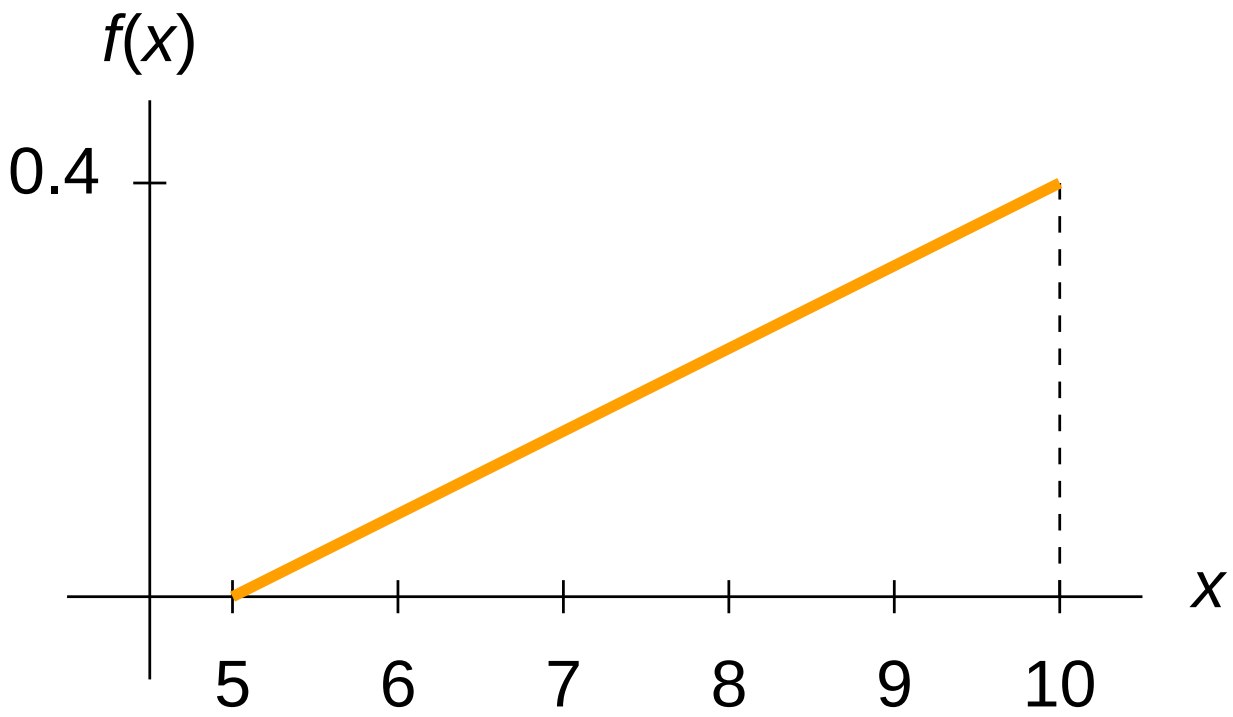


A Non-Uniform Distribution

- What is the probability that $6 \leq X \leq 8$?



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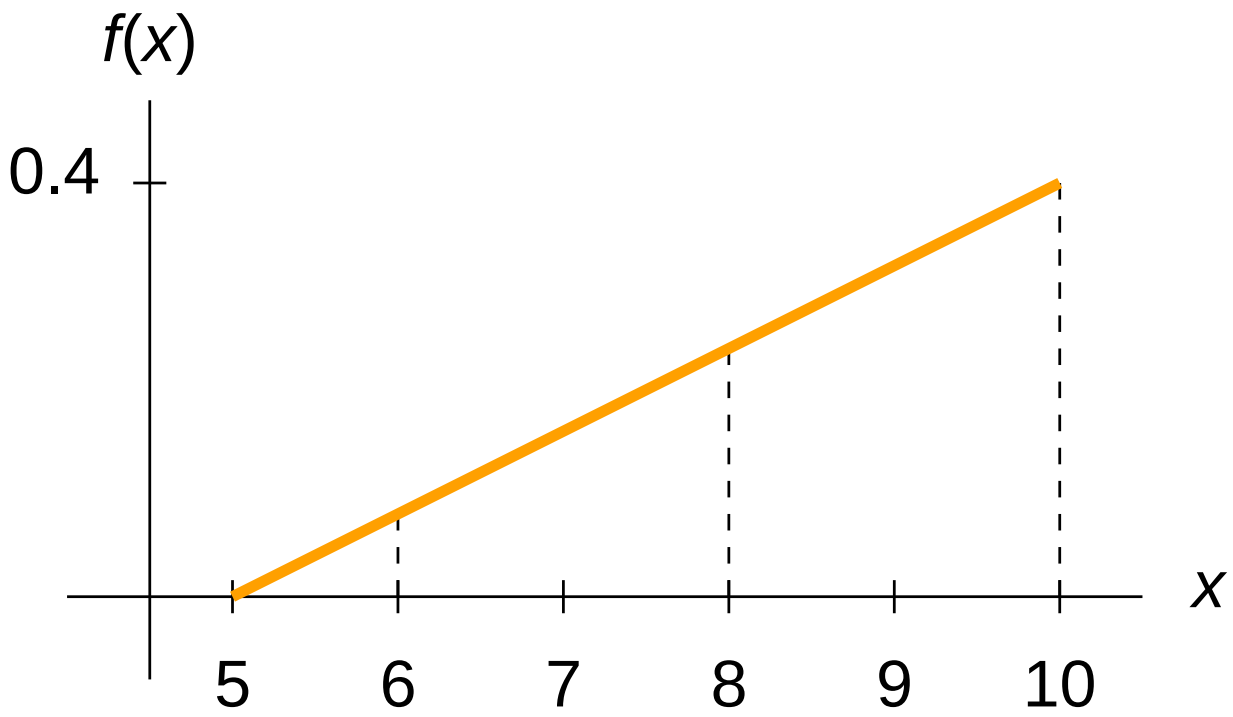

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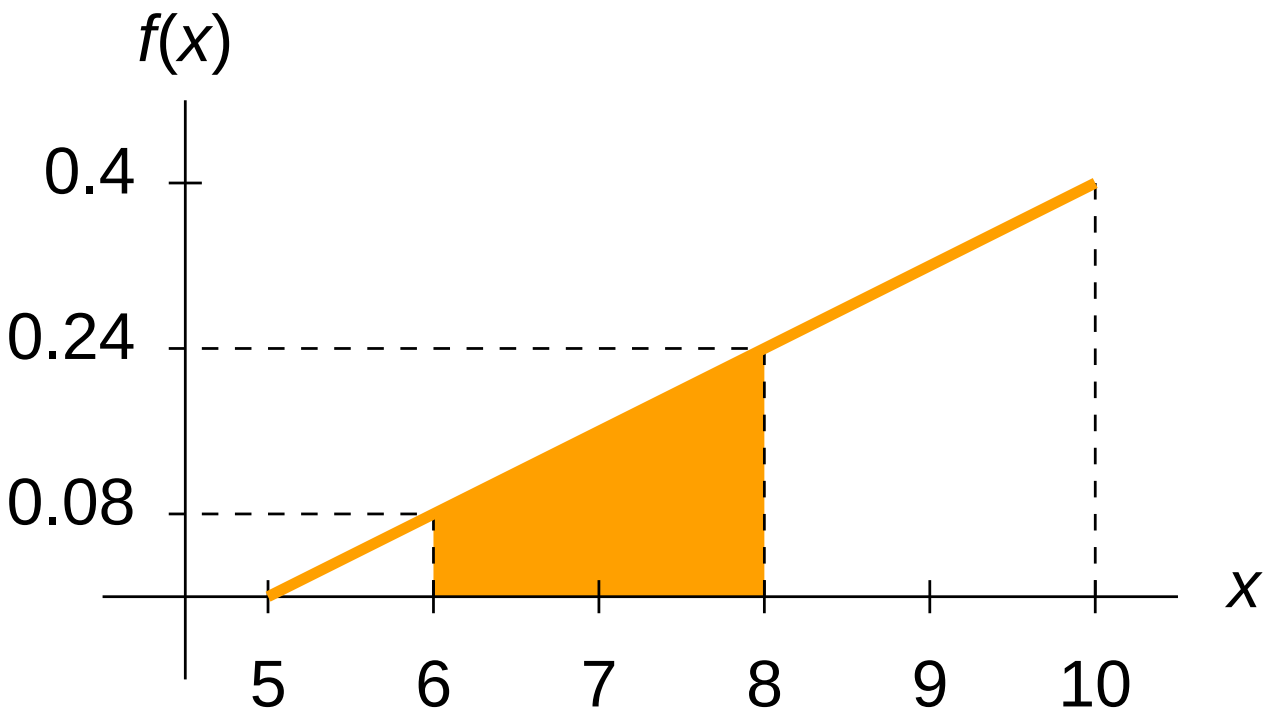

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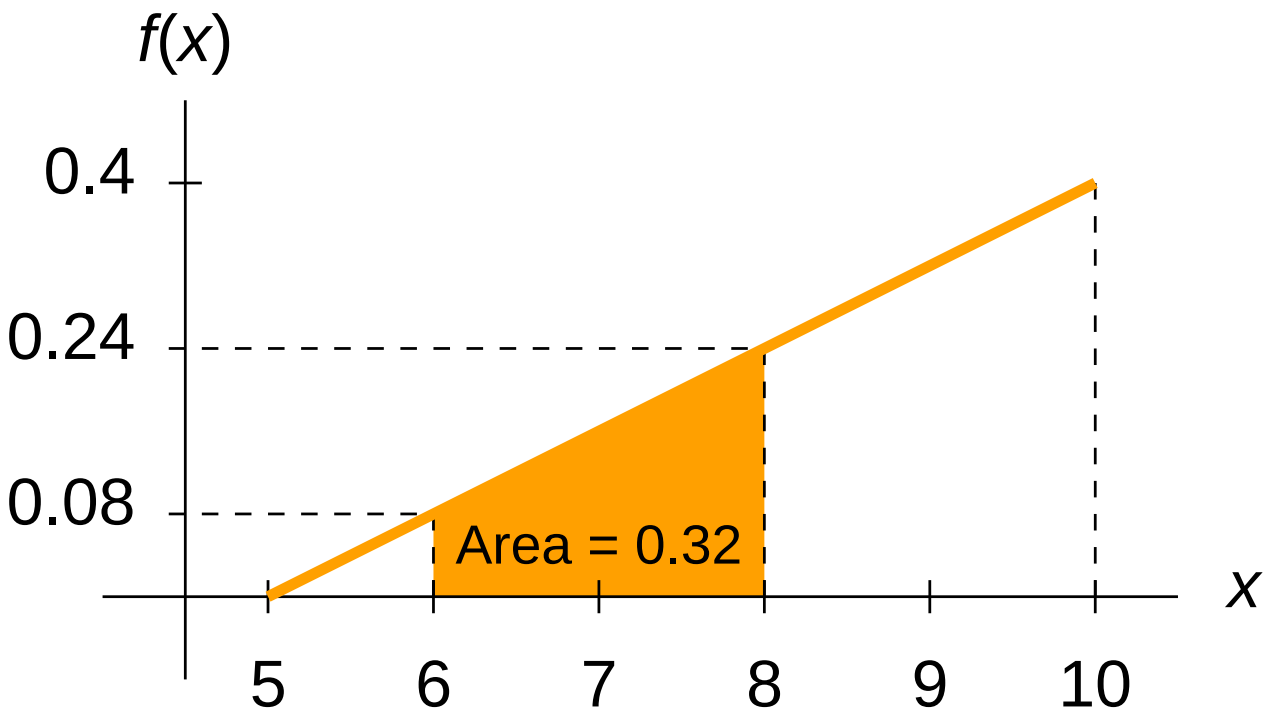
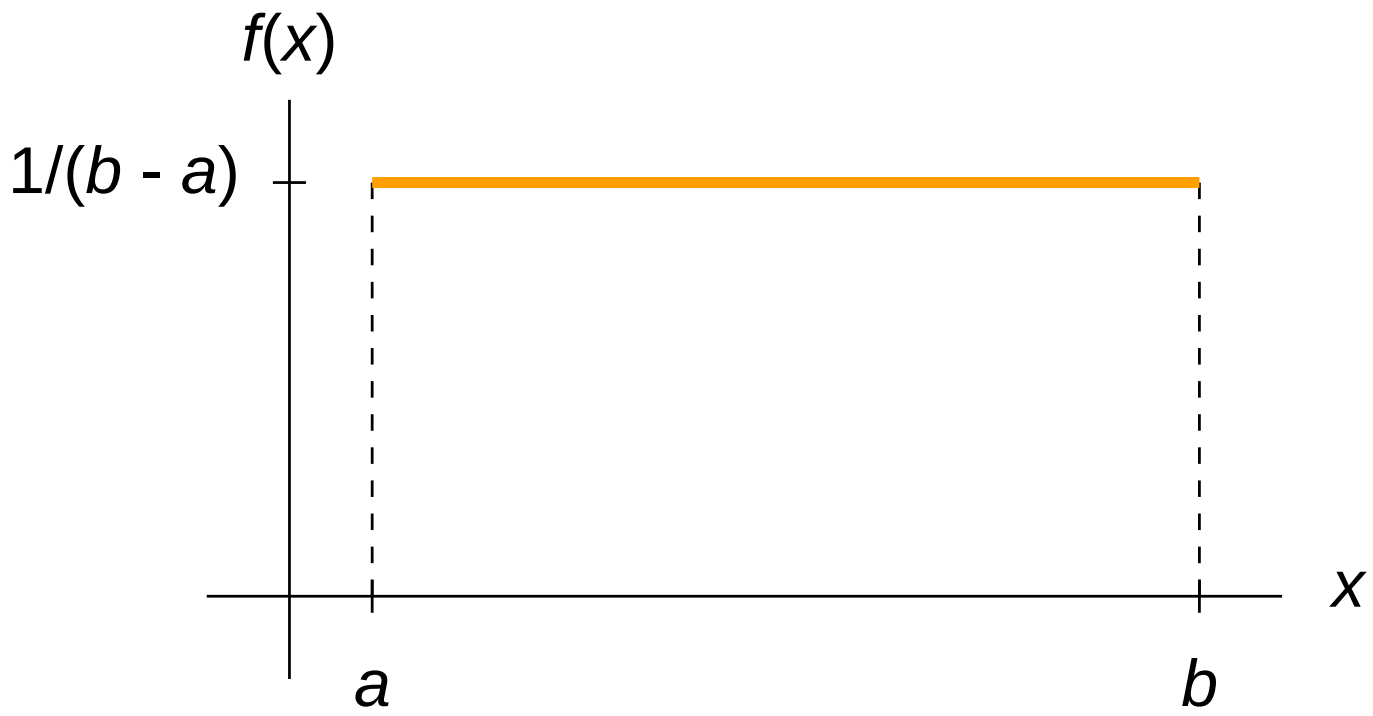


Uniform Distributions

- The uniform distribution from a to b is denoted $U(a, b)$.

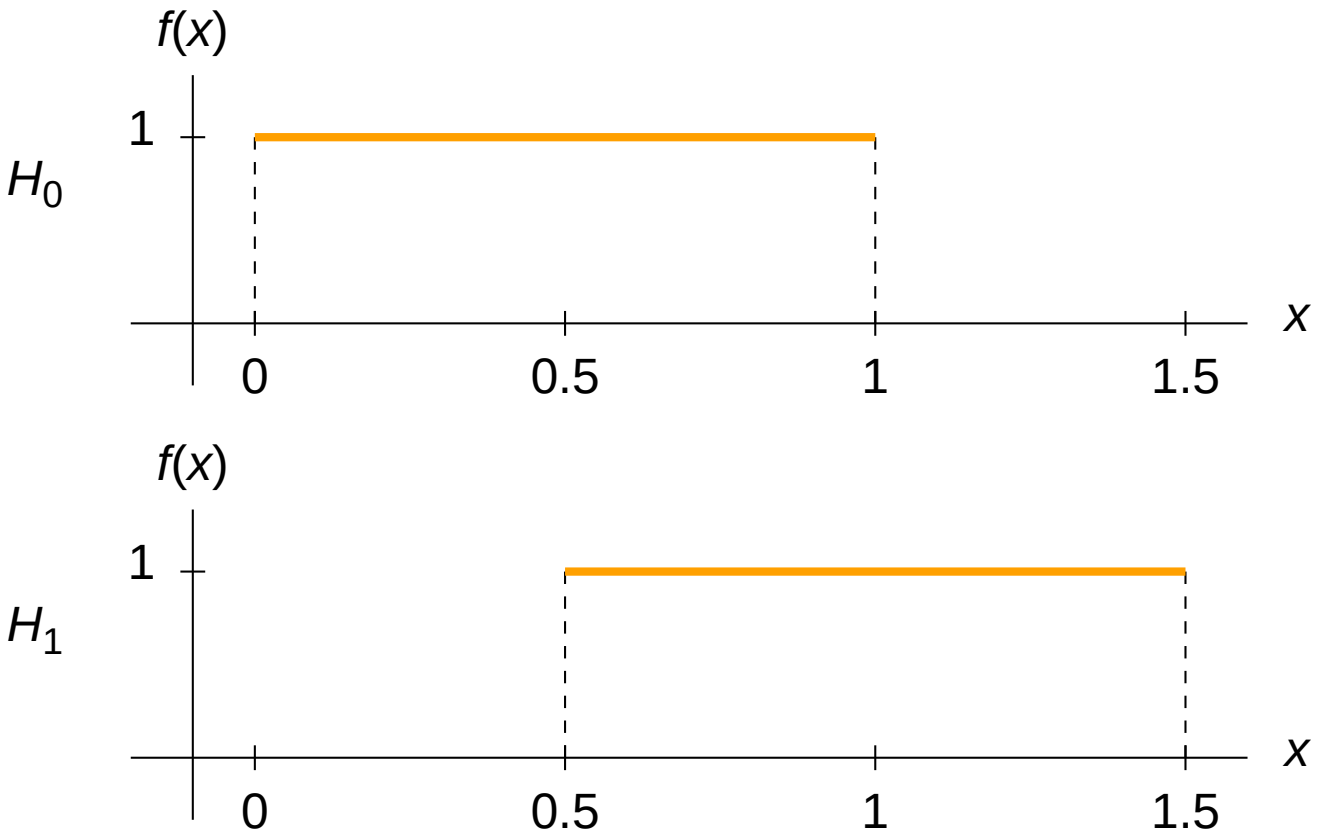


Hypothesis Testing ($n = 1$)

- An experiment is designed to determine whether a random variable X has the distribution $U(0, 1)$ or $U(0.5, 1.5)$.
  - $H_0 : X$ is $U(0, 1)$.
  - $H_1 : X$ is $U(0.5, 1.5)$.
- One value of X is sampled ($n = 1$).
- If X is more than 0.75, then H_0 will be rejected.

Hypothesis Testing ($n = 1$)

- Hypothetical distributions of X under H_0 and H_1:



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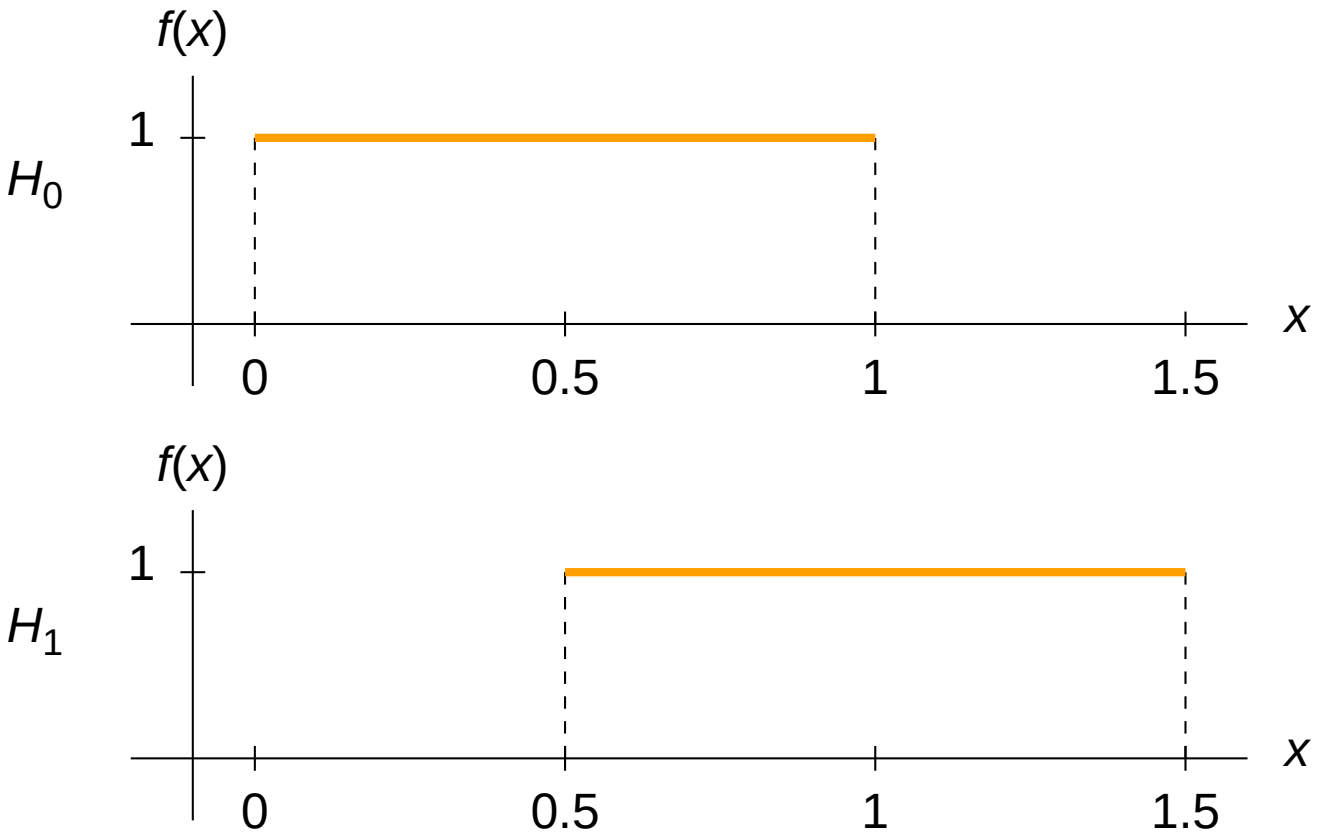

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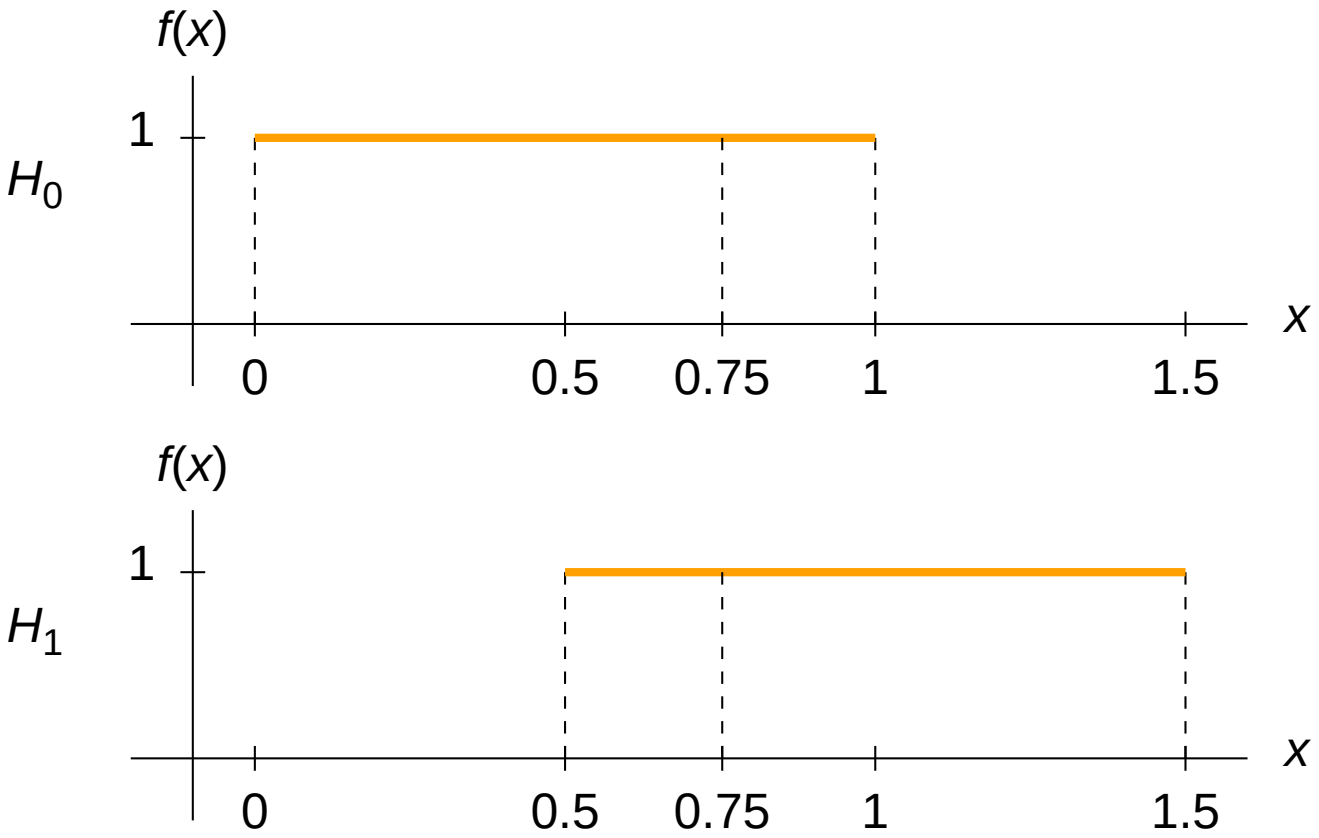

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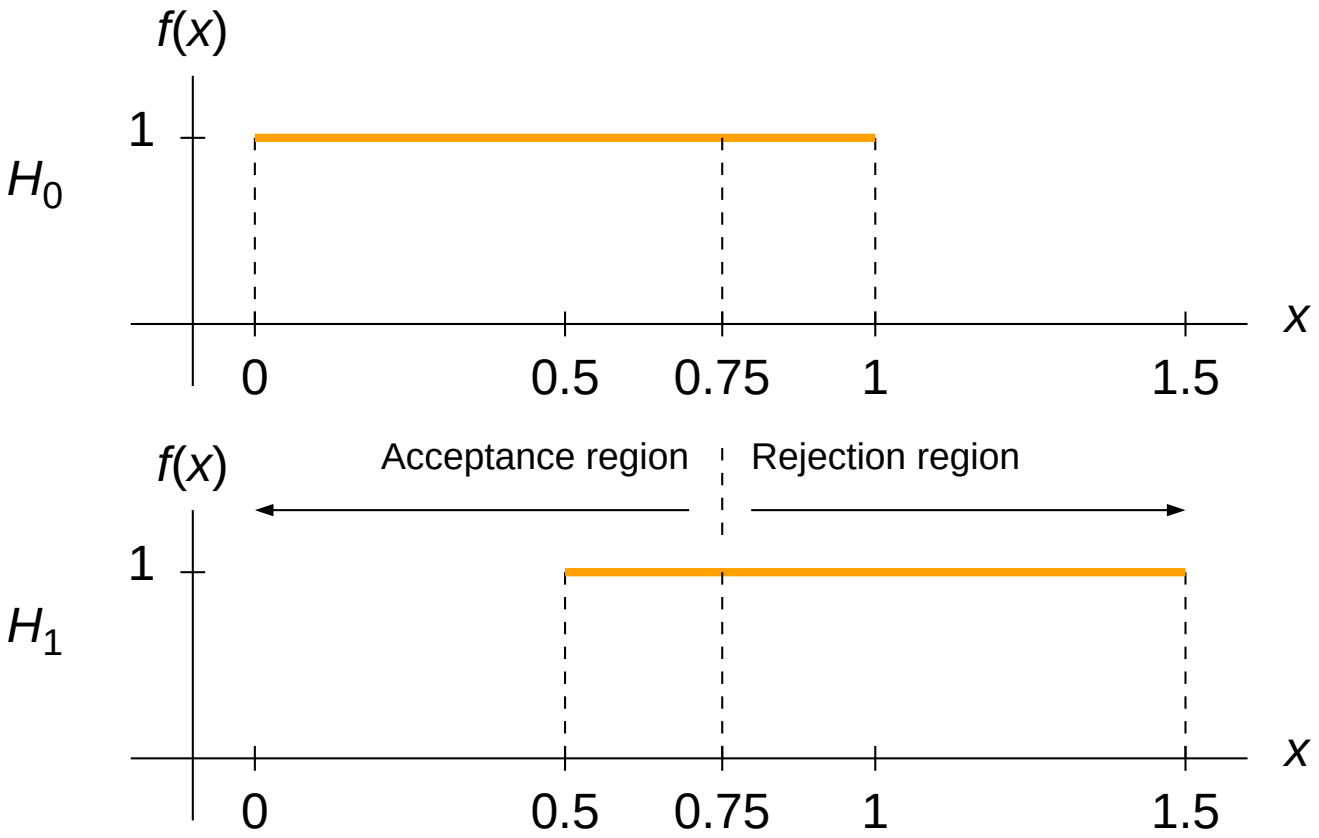

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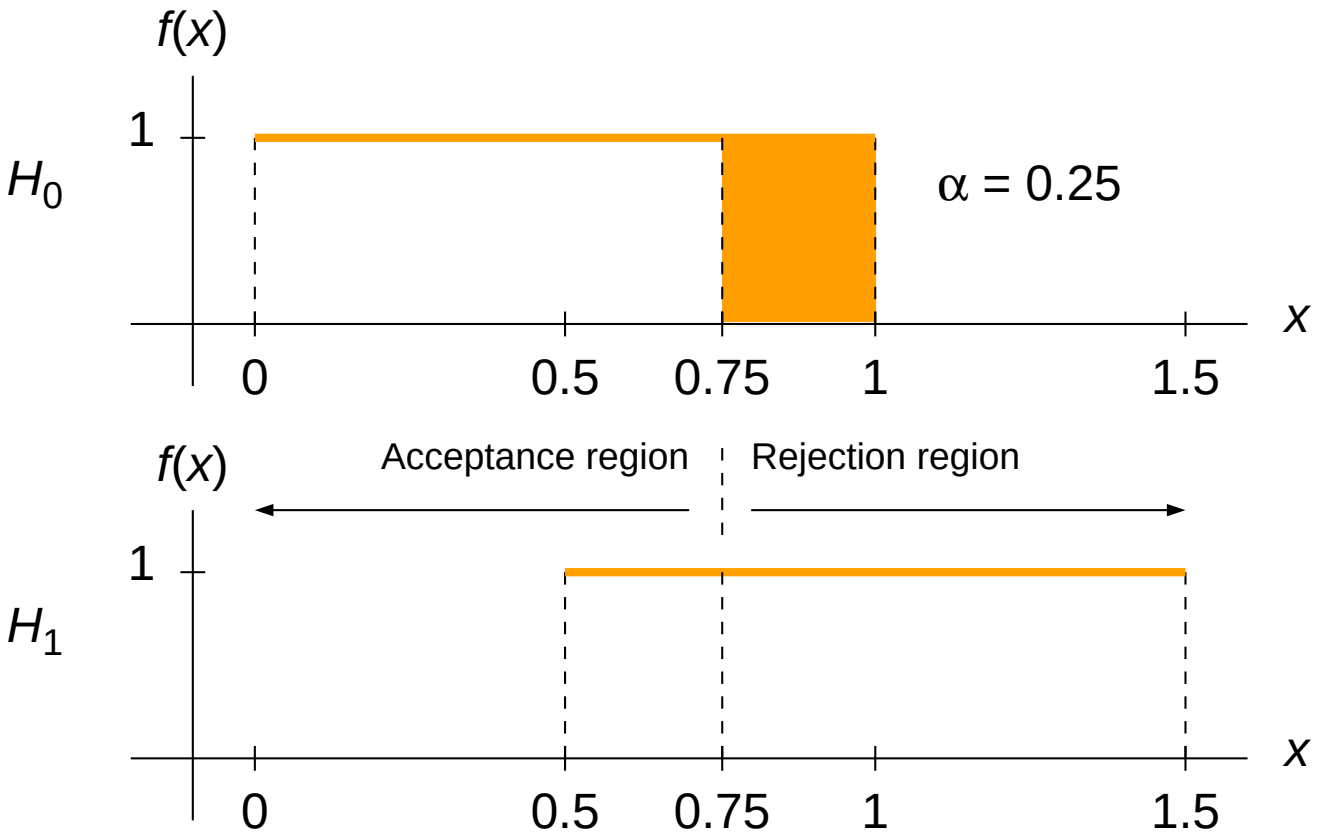

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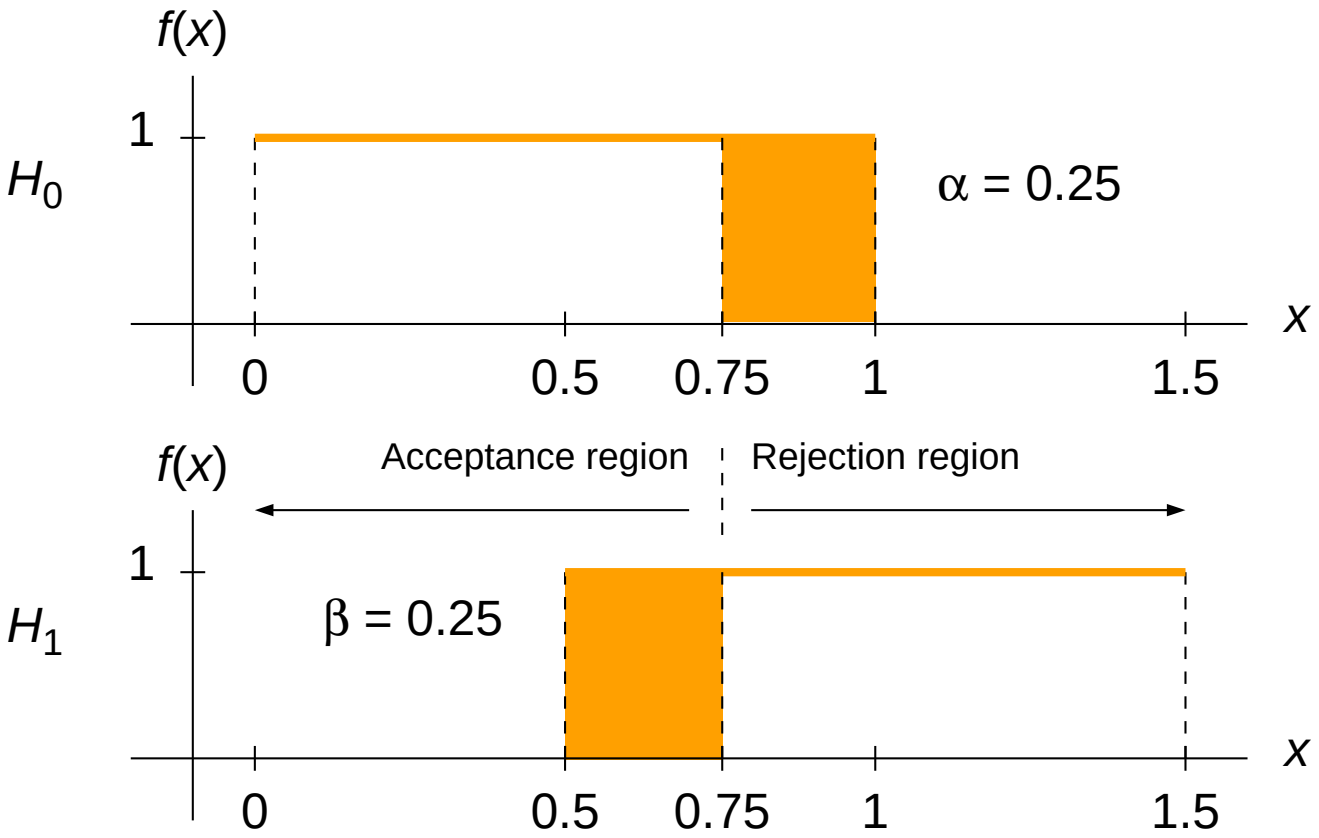
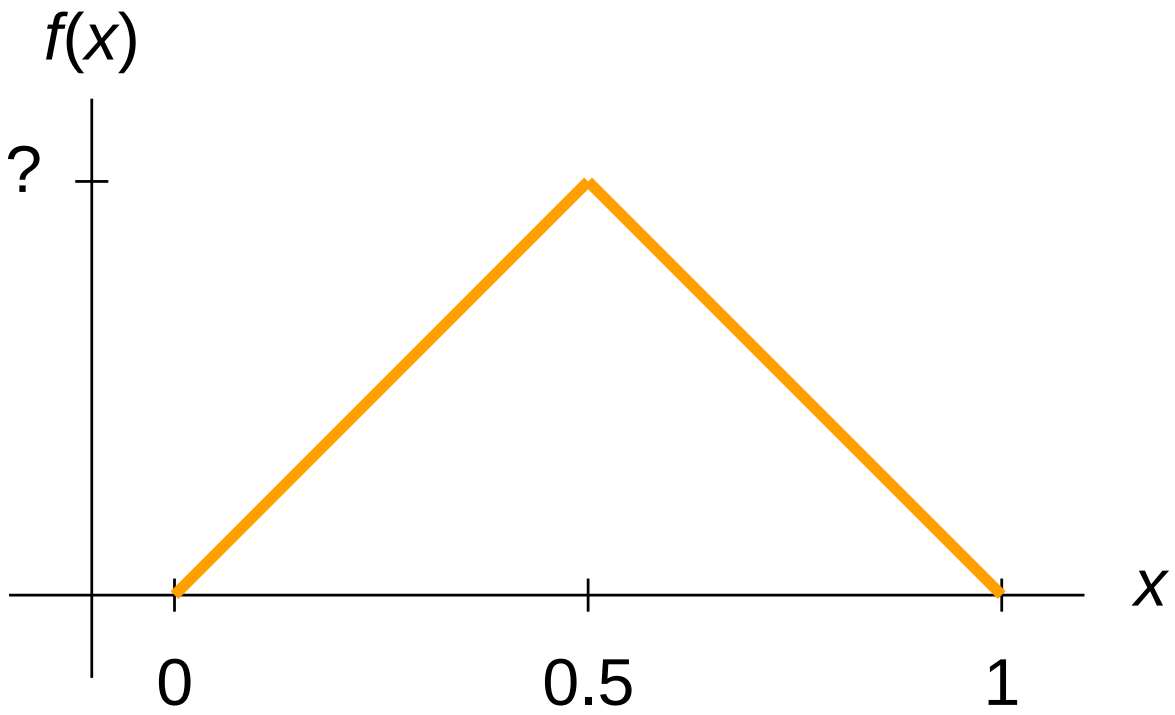


Example

- Now suppose we use the TI-83 to get *two* random numbers from 0 to 1.
- Let $X_2 =$ the average of the two random numbers.
- What is the pdf of X_2?

Example

- The graph of the pdf of X_2.



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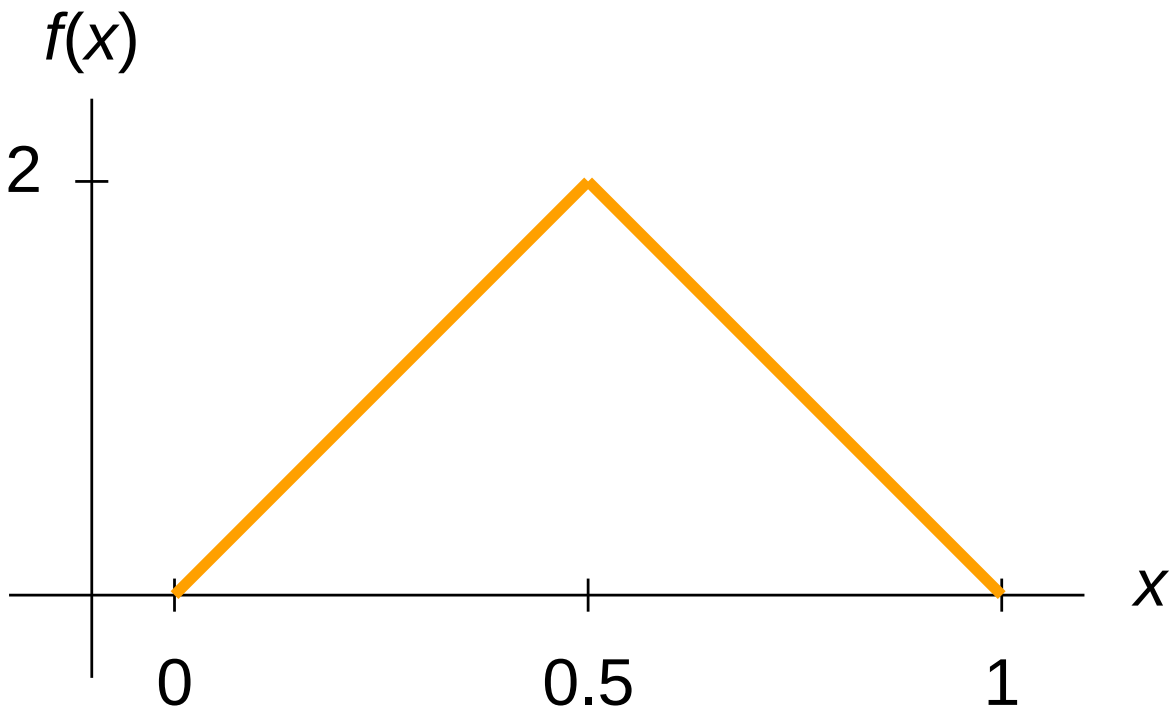

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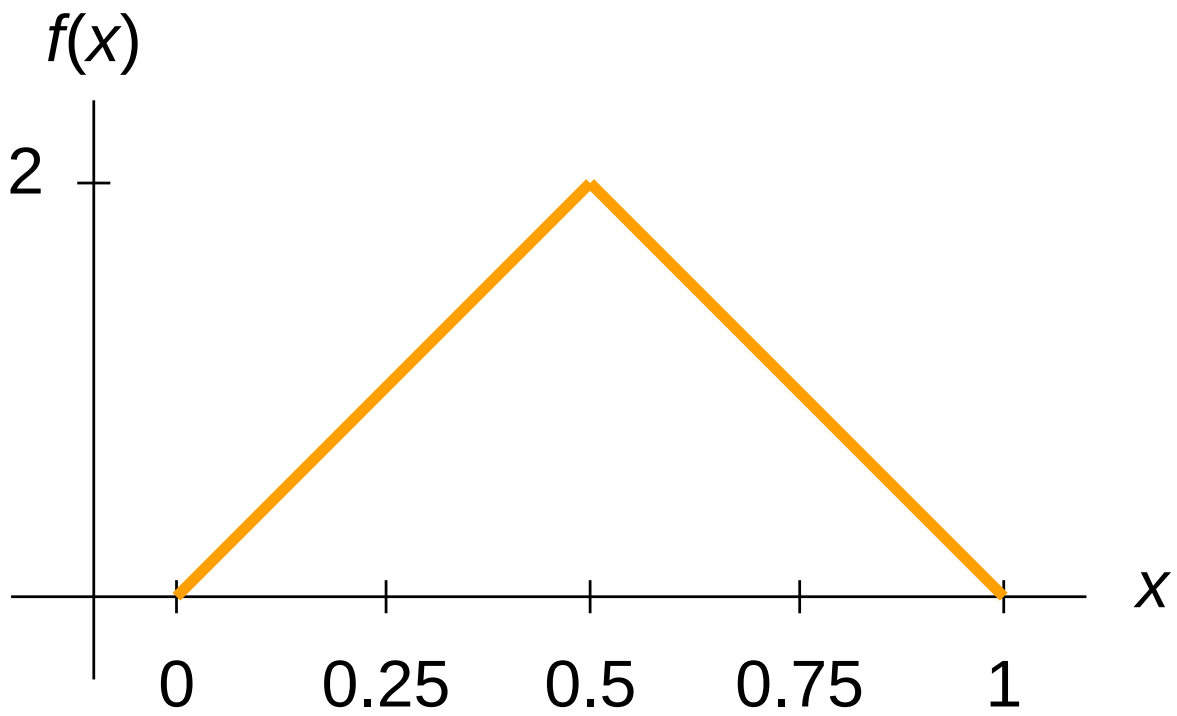

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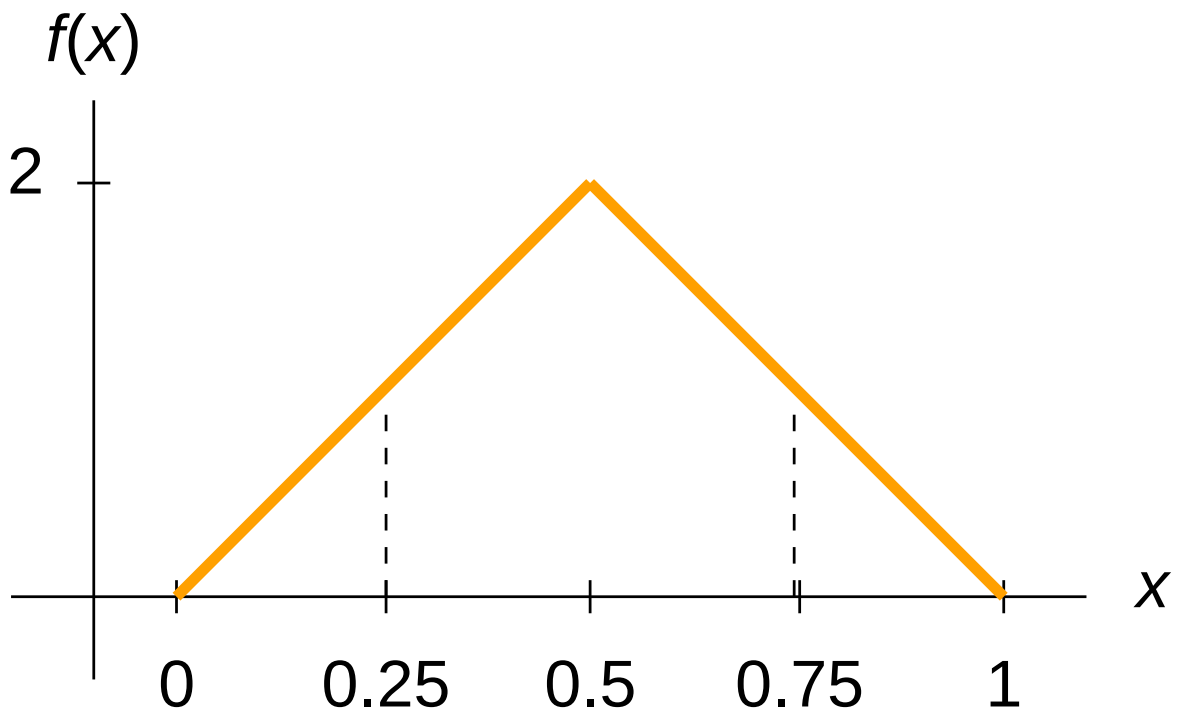

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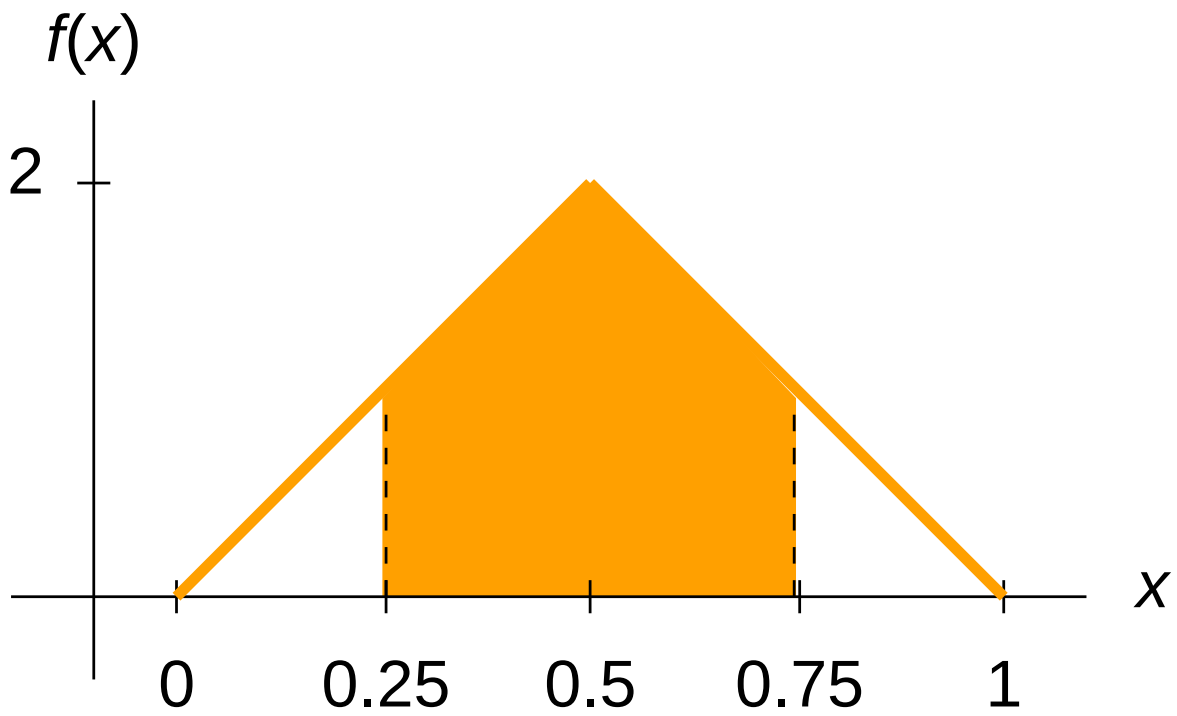

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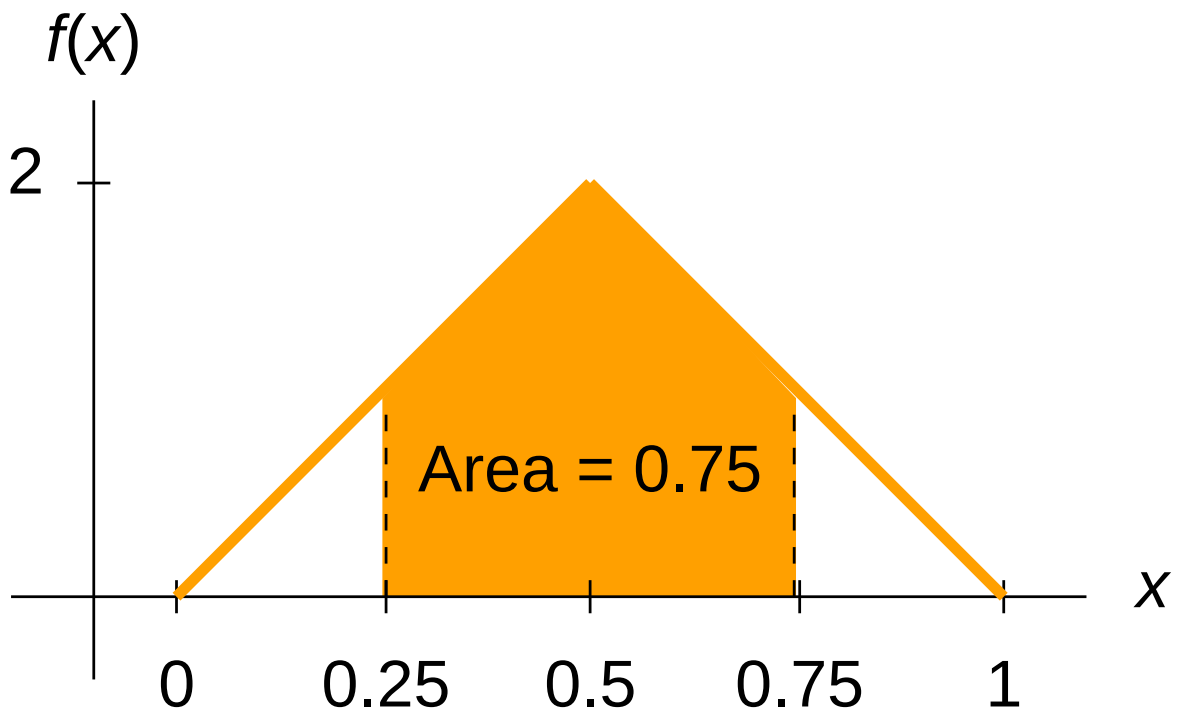

Hypothesis Testing ($n = 2$)

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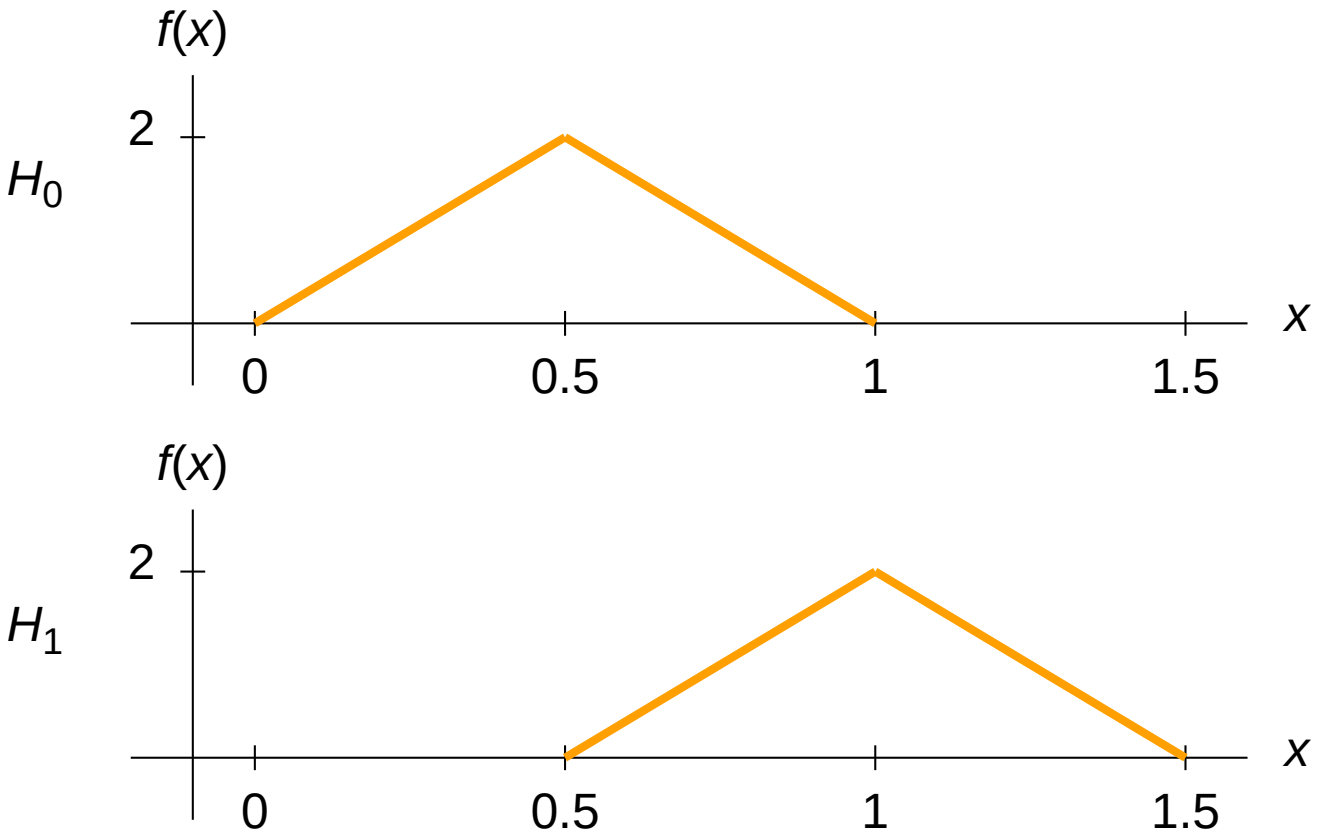
  - $H_0 : X$ is $U(0, 1)$.
  - $H_1 : X$ is $U(0.5, 1.5)$.
- Two values of X are sampled ($n = 2$).
- Let X_2 be the average.
- If X_2 is more than 0.75, then H_0 will be rejected.

Hypothesis Testing ($n = 2$)

- Hypothetical distributions of X under H_0 and H_1:



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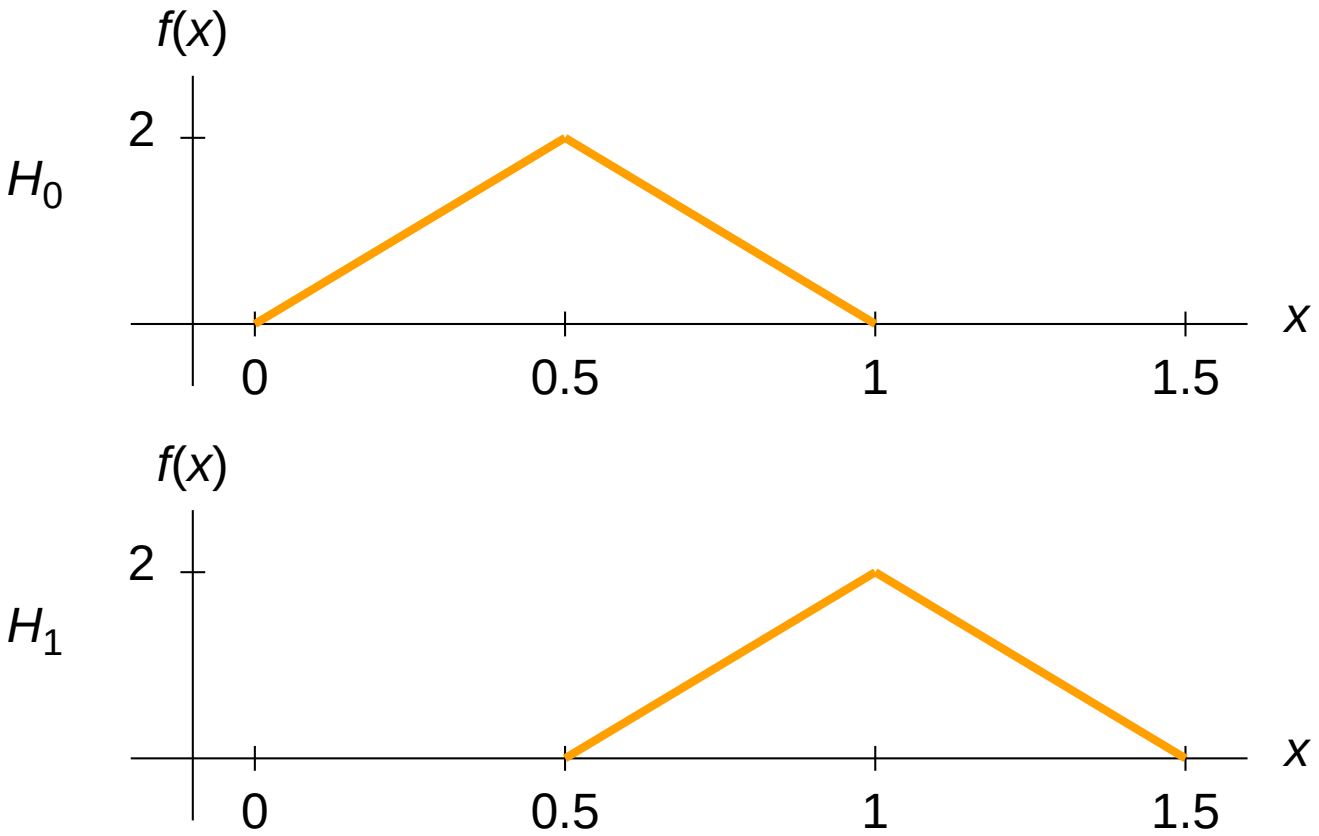

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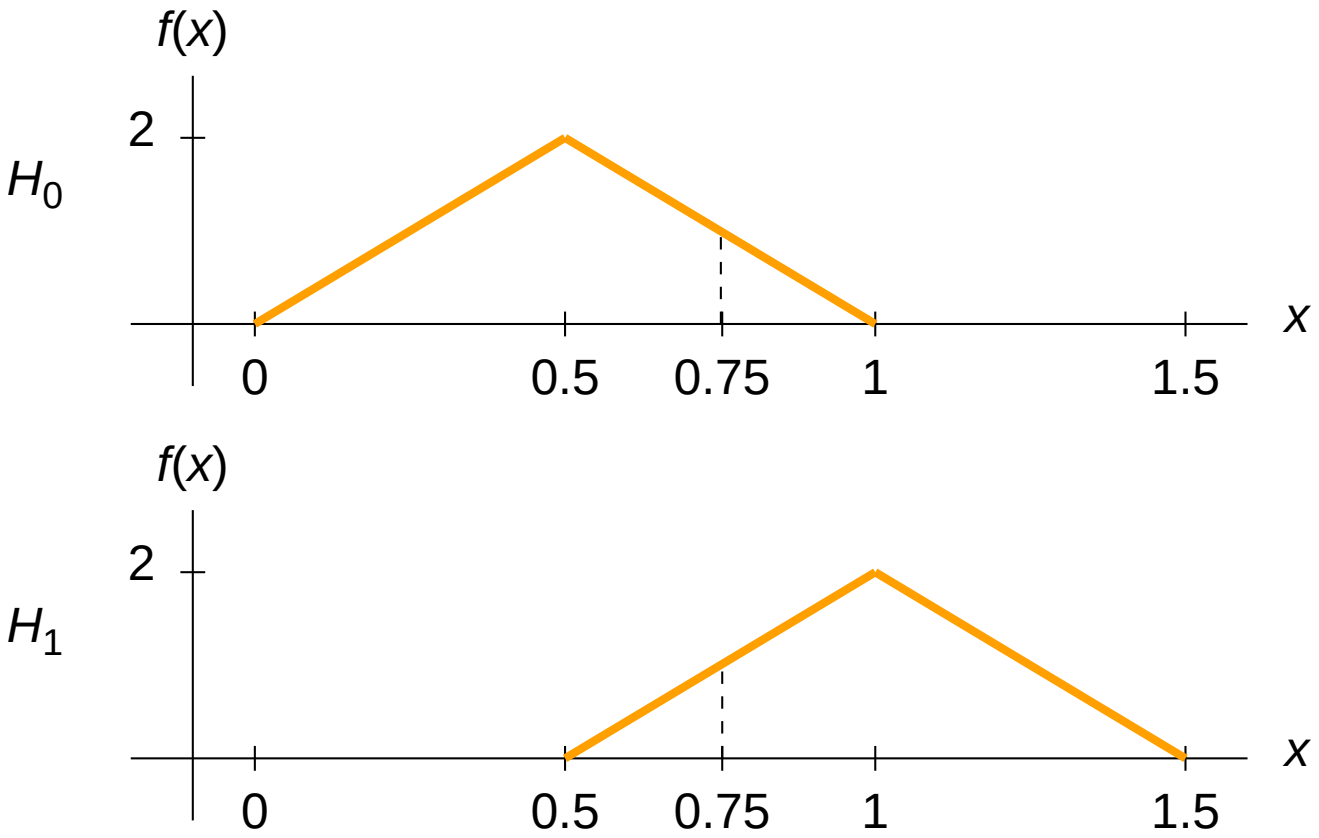

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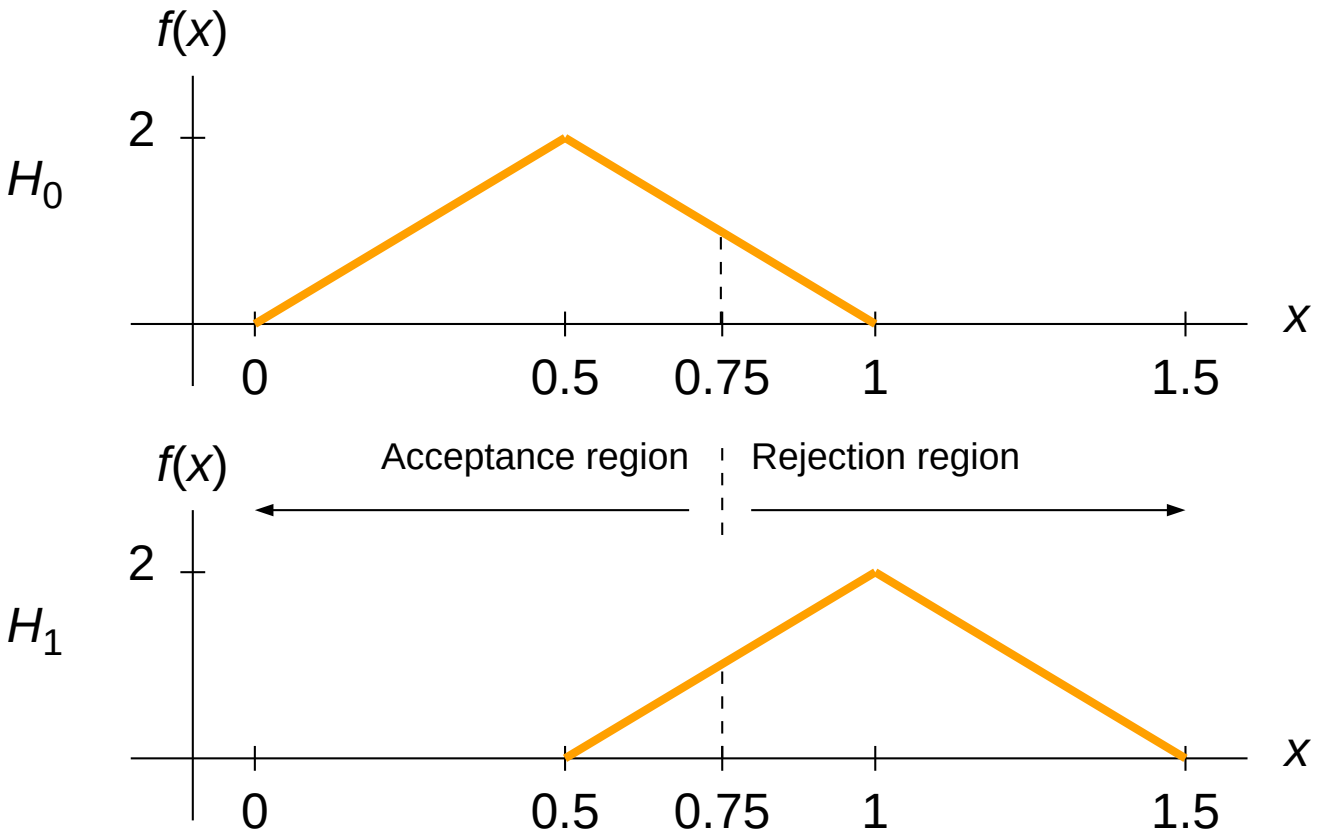

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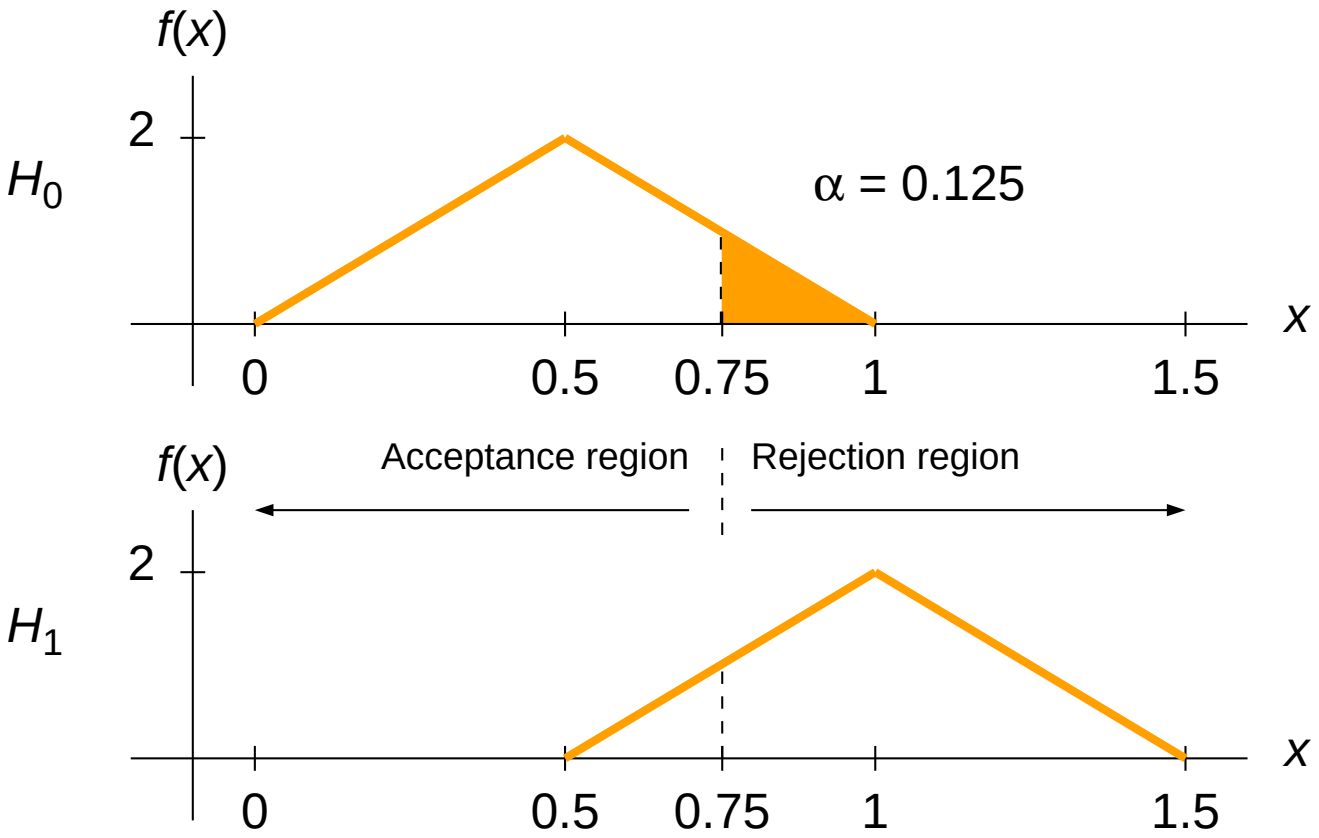

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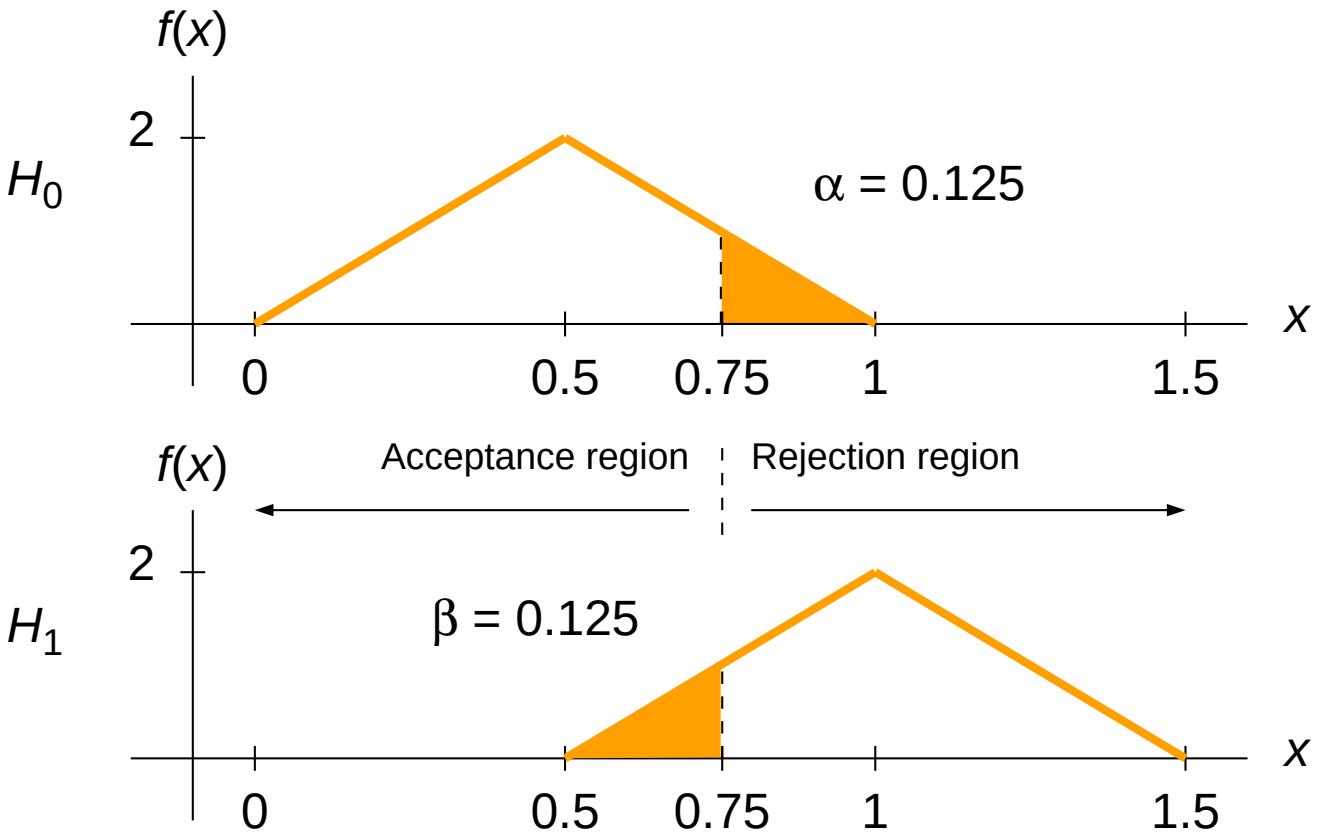

Conclusion

- We may conclude that by increasing the sample size, we can lower both α and β simultaneously.

Assignment

Homework

- Read Section 7.5.4, pages 478 - 481.
- Exercises 64 - 66, 68, 69, page 481.
- Review Exercises 105, 106, page 489.